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THE DERIVATION OF ELECTROMAGNETIC FIELDS FROM A BASIC WAVE-FUNCTION

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1. Derivation of a Logarithmic Wave Function.—Electromagnetic fields may be derived from wave-functions in at least two ways that are analytically distinct.

In the first place four wave-functions satisfying a divergence relation may be chosen as the components of a 4-vector and field-vectors derived from these four electromagnetic potentials in the usual way. The four potentials may in their turn be derived by differential operations from the components of a 6-vector whose components may be taken to be any six wave-functions. This method is a generalization of the well-known methods of Fitzgerald and Hertz;[1] it has the disadvantage that the wave-functions cannot be chosen arbitrarily if magnetic poles are to be excluded.

Secondly, a standard type of definite integral may be adopted for the representation of a wave-function and suitable variations made in the limits and arbitrary functions that will give six wave-functions capable of representing the components of the field-vectors E and H in an electromagnetic field.

The second method has been adopted in only a few cases[2] and there is still much to be learned regarding it. The method will be studied here in connection with a definite integral of type

$$V = \int W f(u) du \qquad (1)$$

in which

$$W = \frac{1}{X^2 + Y^2 + Z^2 - c^2T^2}$$

and $X = x - \xi(u), \quad Y = y - \eta(u).$
$Z = z - \zeta(u), \quad T = t - u.$

The quantity c represents the velocity of light and is supposed to be constant. The function $f(u)$ is arbitrary.

When u is constant the quantity W is a wave-function which we shall regard as basic since it is the analogue of the fundamental potential function $1/r$ of electrostatics.

When ξ, η, ζ and u are all real the function W has singularities spread over the double cone. $X^2 + Y^2 + Z^2 = c^2\,T^2$ which may be represented geometrically in a Minkowski world in which x, y, z and ct are running coordinates. The locus of the singularities will be called the *skeleton* of a wave-function.

A wave-function W without singularities may be obtained by making u complex and complex integration has been used by Conway[3] and Herglotz[4] to derive the Liénard electromagnetic potentials from the basic wave-function. The components of the field vectors in the field of an electric pole have not, however, been represented by contour integrals of the standard type. An attempt will be made here to use standard integrals taken along real paths to represent the components of the field vectors in a fundamental type of electromagnetic field.

If we wish to integrate $W\,f(u)$ along a real path we must clearly avoid values of u for which W is infinite, i.e., for which

$$X^2 + Y^2 + Z^2 = c^2T^2$$

It is known that when $\xi'^2(u) + \eta'^2(u) + \zeta'^2(u) < c^2$ there is one such value of u for which $u \leq t$. We shall denote this value by τ. $\forall$ And so for every $P(x, y, z, t)$ the equation

$$[x - \xi(\tau)]^2 + [y - \eta(\tau)]^2 + [z - \xi(\tau)]^2 = c^2\,[t - \tau]^2 \qquad (1a)$$

defines the corresponding $\tau((x, y, z, t)$.

An integral between constant limits is useless, for the quantity τ depends on x, y, z and t and these quantities may be chosen so as to bring τ within any previously assigned range of integration.

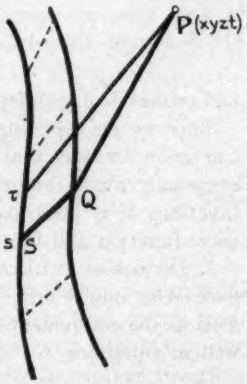
We must work, then, with limits, one of which at least depends on x, y, z and t. Now it has been shown by one of us that the integral (1) is indeed a wave-function if taken between the limits $-\infty$ and s where s is defined by the equations.

$$[x - \xi(s) - l(s)]^2 + [y - \eta(s) - m(s)]^2 + [z - \zeta(s) - n(s)]^2 = c^2\,[t - s - p(s)]^2$$
$$s < s + p(s) < t$$
$$[l(s)]^2 + [m(s)]^2 + [n(s)]^2 = c^2[p(s)]^2$$

and that the dangerous values which of u for W is infinite generally lie outside the range $(-\infty, s)$ though τ may coincide with s in certain cases.

Regarding $\xi(s)$, $\eta(s)$, $\zeta(s)$ as the rectangular coördinates at time s of a moving point S, and $\xi(s) + l(s)$, $\eta(s) + m(s)$, $\zeta(s) + n(s)$ as the coördinates at time $s + p(s)$ of an associated moving point, Q, the quantity s defined by the above equations may be interpreted geometrically as follows:

Let the world lines of S and Q be drawn according to Minkowski's scheme and let P be the world point (x, y, z, t). Let a light-line be drawn from P in the direction in which t decreases to meet the world line of Q and from the point of intersection draw a light-line in the direction in which t decreases so as to meet the world line of S.



The value of t at the point of intersection is the value of s defined by the above equations and this value is unique if the velocities of S and Q are always less than c.

Thus s is defined by means of a broken light-line from P to S and is clearly less than τ, which is defined by means of a direct light-line from P to S.

There is one case, however, when $s = \tau$ and this occurs when P lies on a prolongation of a light-line drawn from S to Q in the direction in which t increases. For such a point P the integral is infinite at the upper limit of the integral and the wave-function V is generally infinite; it may be finite, however, if $f(s) = 0$.

The skeleton of V is thus a "wing" made up of light-lines starting from the world line of Q and proceeding in the direction in which t increases. The breadth of the wing may be made as small as we please by choosing $f(u)$ so that it differs from zero only for a short range of values of u.

Now our object is to carry the limit s close up to the value τ and to obtain logarithmic expressions analogous to those which occur in the theory of Cauchy's principal values of integrals. In order to obtain finite expressions we shall introduce a second point R associated with S, the coördinates of R at time $\sigma + \omega(\sigma)$ being

$$\xi(\sigma) + \lambda(\sigma),\ \eta(\sigma) + \mu(\sigma),\ \zeta(\sigma) + \nu(\sigma)$$

Instead of $-\infty$ we now use a second limit σ defined by the equations

$$[x - \xi(\sigma) - \lambda(\sigma)]^2 + [y - \eta(\sigma) - \mu(\sigma)]^2 + [z - \zeta(\sigma) - \nu(\sigma)]^2 = c^2[t - \sigma - \omega(\sigma)]^2$$

$$\sigma < \sigma + \omega(\sigma) < t$$

$$[\lambda(\sigma)]^2 + [\mu(\sigma)]^2 + [\nu(\sigma)]^2 = c^2[\omega(\sigma)]^2$$

Moving the world lines of Q and R close up to that of S the limiting value of the integral is

$$V \doteq -\frac{f(\tau)}{2M}\log\frac{L}{\Lambda}$$

where

$$\left.\begin{aligned} L &= Xl(\tau) + Ym(\tau) + Zn(\tau) - c^2Tp(\tau) \\ \Lambda &= X\lambda(\tau) + Y\mu(\tau) + Z\nu(\tau) - c^2T\omega(\tau) \\ M &= X\xi'(\tau) + Y\eta'(\tau) + Z\zeta'(\tau) - c^2T \\ X &= x - \xi(\tau). \qquad Y = y - \eta(\tau) \\ Z &= z - \zeta(\tau), \qquad T = t - \tau \end{aligned}\right\} \tag{3}$$

and primes denote differentiations with respect to τ.

Since we are concerned only with the ratios of l, m, n, p, to $\lambda, \mu, \nu, \omega$ we can ignore the fact that in the above construction l, m, n, p. $\lambda, \mu, \nu, \omega$ are eventually made infinitesimally small and we may regard them as finite functions of τ. It is easily verified that the function V thus defined is a wave-function and that its skeleton consists of two wings issuing from S.

2. Derivation of Electromagnetic Fields from the Logarithmic Wave-Function.—Our object now is to derive a set of wave-functions which can be used as the components of two field-vectors E and H satisfying the Maxwellian equations

$$\text{curl } \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \text{ div } \mathbf{E} = o$$

$$\text{curl } \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \text{ div } \mathbf{H} = o$$

In the first place it should be remarked that if E and H are wave-functions of types

$$\mathbf{E} = \mathbf{e}f(\tau), \qquad \mathbf{H} = \mathbf{h}f(\tau) \tag{4}$$

where $\tau(x, y, z, t)$ is defined by equation (1a) and the function $f(\tau)$ can be chosen arbitrarily, the conditions

$$\left.\begin{aligned} \Delta\tau \times \mathbf{h} &= \frac{1}{c}\frac{\partial\tau}{\partial t}\mathbf{e} \qquad & \Delta\tau.\,\mathbf{E} &= o \\ \Delta\tau \times \mathbf{e} &= -\frac{1}{c}\frac{\partial\tau}{\partial t}\mathbf{h} \qquad & \Delta\tau.\,\mathbf{H} &= o \end{aligned}\right\} \tag{5}$$

must be satisfied. These conditions are sufficient to make wave-functions of type (4) solutions of Maxwell's equations provided we add the equations which express that E and H are wave-functions for all forms of the function $f(\tau)$.

The conditions (5) are analogous to the conditions of compatibility which occur in the theory of the propagation of waves of discontinuity.[5] The geometrical meaning of the conditions is that a world-plane through P representing the direction of the 6-vector (e, h) contains the light-line from S and is tangential to the light-cone whose vertex is at S. This indicates a method of obtaining the components of the field-vectors.

Using a geometrical picture in the Minkowski space in which x, y, z and ict are rectangular coördinates of a point we displace the points (l, m, n, icp), $(\lambda, \mu, \nu, ic\omega)$ by means of small rotations through the same angle $\delta\theta$ about the planes of yz, zx, xy, xt, yt and zt in turn. In this way we obtain six components of types

$$H_x = \frac{f(\tau)}{M}\left[\frac{nY - mZ}{L} - \frac{\nu Y - \mu Z}{\Lambda}\right]$$

$$E_x = \frac{cf(\tau)}{M}\left[\frac{pX - lT}{L} - \frac{\omega X - \lambda T}{\Lambda}\right]$$

where X, Y, Z, T have the same values as in (3).

The skeleton of the field thus specified consists of two wings issuing from S and made up of electric charges radiated along light-lines drawn from different positions of S. Whenever a positive charge is radiated in one direction a compensating negative charge is radiated in another direction. By suitably choosing $f(\tau)$ the breadth of each wing may be made effectively as small as we please. In this fundamental type of field each component is a limiting form of a standard wave-function of type

$$\int_{\sigma}^{s} Wf(\tau)d\tau - \int_{\sigma'}^{s'} Wf(\tau)d\tau$$

The field is regarded as fundamental on account of the simplicity of the field-vectors and the atomistic character of its skeleton. Moreover all the fields that are of particular interest in physics may be derived from this type by superposition, the field of an electric pole being obtained by an operation analogous to differentiation in which the wings of two fields cancel one another out.

When $\xi = \eta = \zeta = l = m = \lambda = \mu = o$, $\begin{matrix} n = cp \\ \nu = -c\omega \end{matrix}$ one wing may be cancelled out by means of the operation $\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}$ while both wings may be cancelled out by means of the operator $\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}$ or the equivalent operator $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. The resulting field is that of a Hertzian

dipole whose varying moment arm is in the direction of OZ. When the logarithmic wave-function

$$V = \frac{1}{r} \log \frac{r+z}{r-z}$$

is taken as one component of a Hertzian vector the resulting field has a skeleton composed of two wings but these wings are the loci of radiated dipoles which are gradually extinguished as they travel with velocity c. This gradual extinction may be regarded as the result of a secondary radiation of dipoles. The field is certainly not as simple as the field which is regarded here as fundamental.

[1] For references see H. Bateman, "*Electrical and Optical Wave Motion*," p. 7.

[2] *L. c.*, pp. 12, 113.

[3] *Proc. London Math. Soc.* (Ser. 2), **1** (1903).

[4] *Gött. Nachr.*, (1904).

[5] O. Heaviside, "*Electrical Papers*," Vol. 2, p. 405; A. E. H. Love, *Proc. London Math. Soc.* (Ser. 2), **1** (1903), p. 37; P. Duhem, *Comptes Rendus*, **131** (1900), p. 1171.

THE INFLUENCE ON SECONDARY X-RAY SPECTRA OF PLACING THE TUBE AND RADIATOR IN A BOX

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A number of experiments have been reported recently[1] on the secondary X-ray spectra produced in secondary radiators composed of chemical elements of low atomic weight by the K series lines from a molybdenum target tube. These spectra show strong lines having the wave-lengths of the primary rays and in addition other strong lines having wave-lengths several hundredths of an Ångström longer. The second set of lines of longer wave-lengths seem to occupy about the same positions in the spectrum no matter what the chemical element composing the secondary radiator may be. Their positions are in substantial agreement with Prof. A. H. Compton's interesting theory of the transfer of energy and momentum from radiation to single electrons.

These spectra differ essentially from those obtained in recently described experiments,[2] performed in our X-ray laboratories. In our spectra the line of longer wave-length shifted its position when secondary radiators of different chemical elements were used. In every case its position agreed very well with the idea that the radiation represented by it was produced by the bombardment of the photoelectrons due to the primary rays against

neighboring atoms in the secondary radiator. This radiation we called tertiary radiation. In those experiments in which we used carbon and oxygen as secondary radiators the tertiary radiation corresponded very well in position with the shifted lines reported from other laboratories. There also appeared in the spectra obtained elsewhere some slight evidence of the tertiary radiation that we found in our experiments with chemical elements of higher atomic number than carbon and oxygen. This tertiary radiation, however, appeared to be very much weaker than in our experiments.

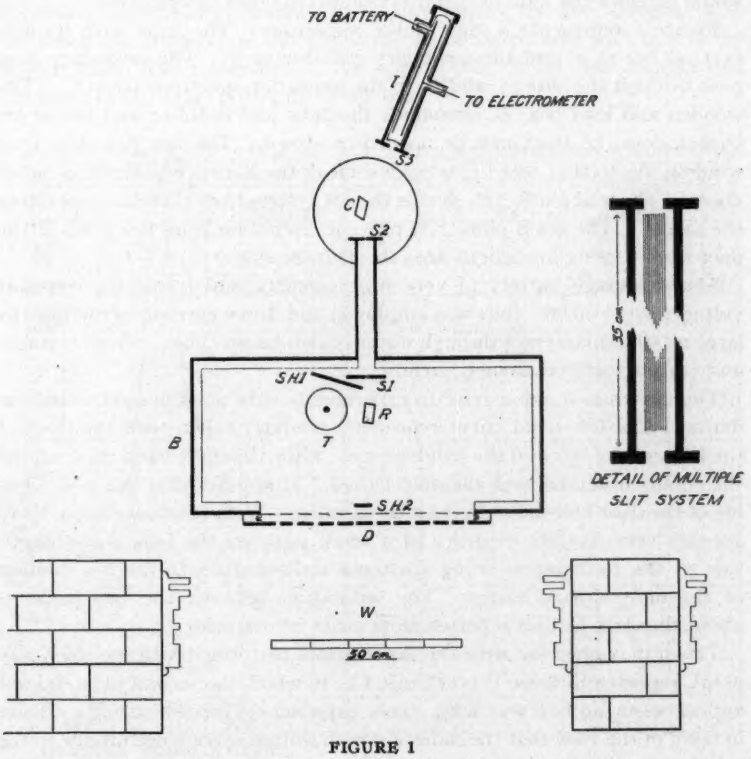

FIGURE 1

On thinking over possible differences between the methods of investigating spectra employed in other laboratories and those used in our own laboratory it occurred to us that perhaps the most important difference lay in the fact that we have never placed our X-ray tubes and radiators in small boxes, whereas others may have used lead boxes with wood or other insulating material composed of carbon and oxygen inside of them. As

has been repeatedly stated in the descriptions of our investigations of X-ray spectra, our X-ray tube and radiator lie in one large room and the X-rays to be examined pass through a hole in the wall to the spectrometer in an adjoining room.

Having just completed a new X-ray plant, designed for researches in another field of X-radiation, in which a wooden box covered with a heavy coating of lead $^1/_4$ inch thick surrounds the X-ray tube, we decided to try some experiments on the secondary radiation from chemical elements such as aluminium and sulphur to see whether such an arrangement of apparatus would produce the kind of spectra obtained in other laboratories.

Figure 1 represents a plan of the apparatus The tube with its axis vertical lies at T and the secondary radiator at R. The secondary rays pass through the slits S_1, and S_2 to the ionization spectrometer at C. The wooden and lead box, B, surrounds the tube and radiator and has at its back a door, D, that may be opened or closed. The box lies close to a window, W, so that when D is open some of the X-rays pass directly out of doors. The lead plate, SH_1 shields the slit system from the direct rays from the target. The brass plate SH_2 prevents radiation from the wood of the door from passing in straight lines through the slits.

A new storage battery of very high capacity and furnishing constant voltages up to 50,000 volts was employed and drove currents sometimes as large as 40 milliamperes through water-cooled target tubes. Both voltages and currents were extremely steady.

The curves in figure 2 refer to experiments with a sulfur secondary radiator. The left hand curve represents readings taken with the door at the back of the box and the window open, while the right hand curve shows the results obtained with the door closed. It appears that the mere closing of the door had some effect on the spectra. With the door closed, there appears very distinct evidence of a small peak on the long wave-length side of the peak representing scattered radiation due to the $K\alpha$ doublet of the molybdenum target. The separation between the two peaks is about the same as that reported from other laboratories.

Taken in connection with the experiments performed with the old X-ray plant, reported in these Proceedings, l.c., in which the second peak did not appear when no box was used, these experiments furnish strong evidence in favor of the view that the radiation with shifted wave-length is due to the box surrounding the tube and radiator and is not characteristic of the secondary-tertiary radiation of the radiator itself.

Subsequent experiments in which a Soller multiple slit system represented in figure 1, was substituted for slits S_1 and S_2, also indicated the presence of the shifted line and emphasized the fact that with this particular arrangement of apparatus the ionization currents obtained with the box closed are somewhat smaller than those obtained under identical

conditions except with the box open. Whether or not this may be entirely ascribed to some stray radiation that did not pass out of the open window has not been definitely determined.

In these experiments the intensity of the radiation of shifted wave-length appears to be very much less than that of the truly scattered radiation. In the experiments reported in the following note, however, the intensities are reversed, the shifted line having the greater intensity. In our experiments a heavy lead shield surrounded the wooden box, whereas in the other experiments lead only partially surrounded the box. It would seem, therefore, that the large amount of lead surrounding the wooden box greatly enhanced the intensity of the unshifted line in comparison with that of the shifted line. The intensities of the lines appear to be very complicated functions of the exact structure of the box.

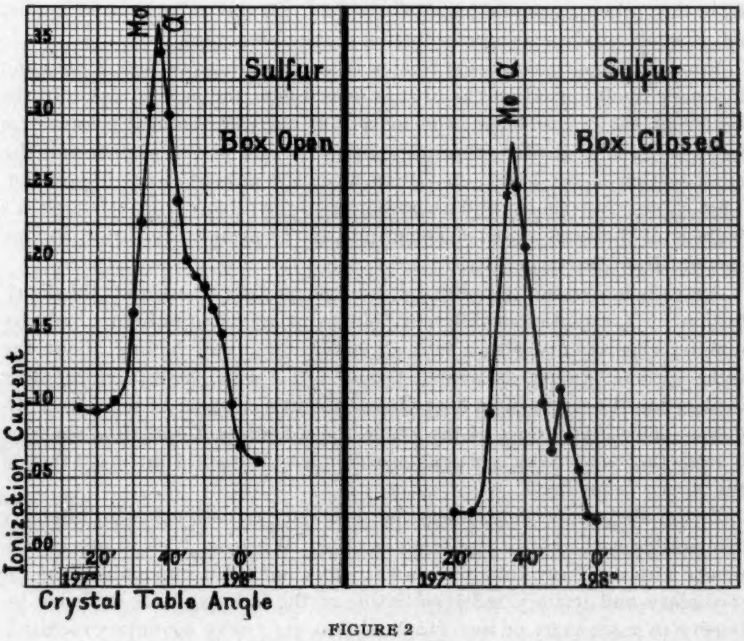

FIGURE 2

With the tube and secondary radiator in a heavy box, the tertiary radiation characteristic of the chemical elements in the radiator is relatively very weak and may be almost invisible.

No one can doubt that a considerable amount of X-radiation must be passing in all directions through a box of the kind described, containing an X-ray

tube in action. This radiation must have the wave-lengths characteristic of the secondary and teritary radiation belonging to the chemical elements composing the walls of the box and whatever there is inside of it. The amount of this box radiation that passes out through the slits into the spectrometer appears to be sufficient to dominate more or less completely the spectrum obtained. At first sight it may seem strange that the box radiation could have sufficient intensity to do this. The following facts, however, give one an idea of the magnitude of secondary radiation in certain cases.

When a roentgenologist takes a photograph of the body of a patient for diagnostic purposes, he uses a device known as the Potter-Bucky diaphragm. This consists of a number of nearly parallel lead strips, moved across between the patient and the photographic plants during the exposure. The purpose of the lead strips is to cut off the secondary radiation coming from the various tissues in the patient's body. If this radiation were not cut off, it would produce a certain blurring of the photograph.

Further, if a patient is being treated by means of high voltage X-rays and if a very small ionization chamber is placed near the patient's skin in the center of the X-ray beam, the ionization current obtained is anywhere from 10 to 50% greater (depending upon the size of the patient and the cross section of the beam, etc.) than when the patient is not in position. This shows that the secondary radiation thrown back from the patient's body is of the same order of magnitude as the primary radiation coming straight from the target itself.

Again it is common practice for roentgenologists to estimate the distribution of X-radiation at different points throughout a patient's body by making experiments with a tank of water, the water having about the same specific gravity as human tissues. If a small ionization chamber is placed in the center of an X-ray beam and at a distance of some 10 centimeters below the surface of the water, the secondary X-radiation reaching it from the surrounding portions of water may exceed in intensity that of the direct beam itself.

In the above mentioned cases the secondary radiation has an intensity comparable with that of the *primary* rays. When we turn to our experiments with an X-ray tube in a box, it does not seem impossible that the secondary and tertiary radiation inside of the box may have sufficient intensity to mask more or less completely the very weak secondary radiation coming from a small secondary radiator inside of the box.

It appears from our experiments and those reported in an accompanying note, (These PROCEEDINGS, Sept. 1924, p. 379) that the X-rays coming through the spectrometer's slits when a box surrounds the tube and secondary radiator, do not represent the true secondary-tertiary spectrum characteristic of the chemical elements composing the secondary radiator.

[1] Compton and Woo, These PROCEEDINGS, June 1924; Ross, These PROCEEDINGS, July 1924; Becker, These PROCEEDINGS, August 1924.

[2] These PROCEEDINGS, **9**, 413, 419 (1923); **10**, 41, 92, 148, 191 (1924).

THE INFLUENCE ON SECONDARY X-RAY SPECTRA OF PLACING THE TUBE AND RADIATOR IN A BOX

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A number of experiments performed in this laboratory have been described[2] in which the secondary and tertiary radiation coming from secondary radiators and due to primary X-rays have been examined. The primary X-rays employed consisted of the K series lines of tungsten and molybdenum and the L series lines of tungsten. Their wave-lengths, therefore, lay in the range from .18 Ångström to 1.48 Ångström. The secondary radiators consisted of chemical elements ranging in atomic number from that of lithium, $N=3$, to that of neodymium, $N=60$. In addition to the scattered radiation, having the wave-lengths of the primary rays and the fluorescent radiation, having the wave-lengths of the spectra characteristic of the chemical elements in the secondary radiators, these spectra contained bands of radiation the short wave-length limits of which corresponded with the wave-lengths calculated on the theory that they consist of tertiary radiation due to the impacts of the photoelectrons against neighboring atoms. Further, the points of maximum intensity in the bands shifted back and forth with the angle between the primary and secondary beams. No other radiation comparable in intensity with the above appeared in the spectra. In particular, no rays appeared having wave-lengths about .024 Ångström longer than the primary wave-lengths, as demanded by Prof. A. H. Compton's theory of the transfer of radiation momentum and energy to single electrons, except in a few cases, in which the tertiary radiation theory predicts that rays of those wave-lengths should be present.

A number of experiments performed in other laboratories have been reported in which the secondary radiation spectra contained not only the scattered radiation of equal wave-lengths with those of the primary rays but also radiation having somewhat longer wave-lengths in substantial agreement with Prof. Compton's theory. In many of these experiments the primary rays came from a molybdenum target tube and fell on a secondary radiator composed of carbon atoms. It is important to notice that the above-mentioned experiments performed in our laboratory indicate

that the K series lines of molybdenum produce in a carbon secondary radiator tertiary rays having wave-lengths corresponding very closely with the wave-lengths calculated on Compton's theory. In other experiments,[3] however, reported recently, radiation has been found, having wave-lengths agreeing with the Compton theory but not occupying the position in the spectrum that would be predicted by the tertiary ray idea.

The question now arises, what, if any, fundamental difference exists between the methods of investigating the spectra employed in our laboratory and those employed elsewhere. In all the experiments mentioned above the Bragg method of producing spectra by reflection by a crystal (usually calcite) has been employed. In our experiments, (as has been stated, l.c.) the X-ray tube and secondary radiator were in a large room, supported close to one of its walls and the X-rays to be examined passed through a hole in the wall into an adjoining room where the spectrometer lay. The room containing the X-ray tube and secondary radiator had large openings in it leading out into a much larger space—a kind of attic on the top floor of our laboratory building. Thus, the part of the X-radiation that did not pass through the hole for spectrometric examination had plenty of opportunity to get away from the neighborhood of the secondary radiator. It appears, however, that in the experiments performed in other laboratories the X-ray tube and secondary radiator lay in small boxes, usually of lead, and often containing wood or other insulating material, largely composed of carbon and oxygen. A small box of this kind surrounding an X-ray tube in action contains a certain amount of radiation traveling through it in all directions and coming from the walls of the box and from whatever there may be inside of it. This radiation takes on the character of the secondary and tertiary radiation corresponding to the chemical elements composing the walls of the box and the objects inside of it. Thinking that perhaps enough of this radiation might stream through the slits of the spectrometer to markedly influence the observed spectrum, we decided to surround our tube and secondary radiator with a box and try experiments more closely analogous to those performed elsewhere. We had a wooden box constructed in sections, so that it could be placed around the tube and secondary radiator and removed again, without otherwise changing the apparatus. We have found that the spectra obtained with the box in place differ essentially from those without the box.

The diagram (fig. 1) represents the plan of our apparatus. A commercial water-cooled molybdenum-target tube lies at A. B represents the secondary radiator. The rays from the secondary radiator, passing through the slits, C and D, strike a calcite crystal, E, belonging to the X-ray spectrometer. F represents the ionization chamber, well protected from stray radiation by lead, the ionization currents being measured by a quadrant electrometer not shown in the diagram. For additional protec-

tion against X-rays, a sheet of lead, G was fastened against the wall.[4] The dotted lines H represent the wooden box that could be placed in position or removed at will. In order that no rays, coming directly from wood, might pass straight through the slits into the spectrometer, a thick sheet of lead was fastened against the back wall of the box in line with the slits, as represented at I in the figure.

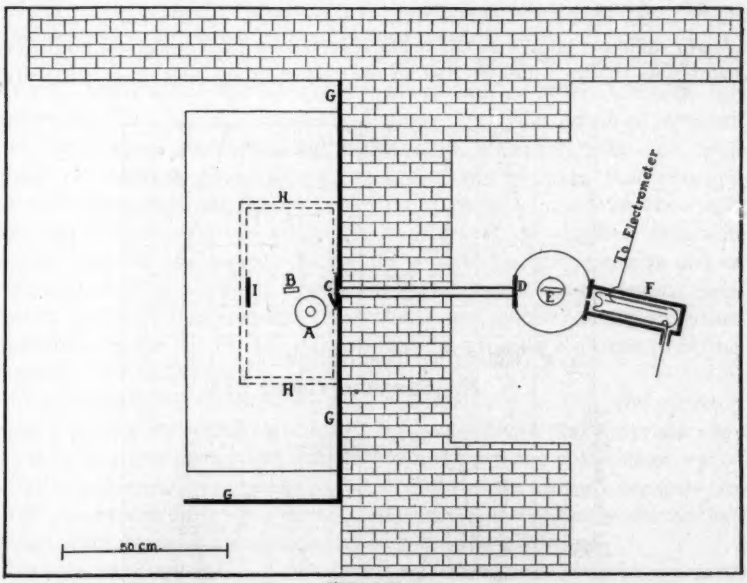

FIGURE 1

In one of these experiments a current of 20 milliamperes passed through the tube from a new storage battery of very high capacity. The constant voltage applied to the terminals of the X-ray tube amounted to 47,000 volts. A block of sulfur, on a light support, served as the secondary radiator. The tertiary radiation from sulfur lies at some distance (more than 1°) from the portion of the spectrum under discussion in this note.

The curves in figure 2 represent the ionization currents as functions of the angles read on the scale that fixes the position of the reflecting crystal. Readings were taken on both sides of the zero line of the spectrometer, an important procedure in measurements to obtain accurate estimates of wavelengths.

The upper curves represent an experiment with the wooden box placed around the X-ray tube and secondary radiator. On each side of the zero, two peaks appear, one of which represents X-rays having precisely the wave-

length of the $K\alpha$ doublet in the primary radiation coming from the molybdenum target. The other represents X-rays having wave-lengths shifted toward larger values. The spectrometer used in these experiments was originally designed for the accurate measurements of wave-lengths in spectra of considerable intensity, and not for the investigation of very weak radiation. Owing to the fact that a brick wall intervenes between the two slits, the distance from the secondary radiator to the calcite crystal is quite large, about 75 centimeters. Under these conditions, in order to obtain sufficient energy in the beam reflected by the crystal, without over loading the X-ray tube, we had to use rather broad slits (each slit width

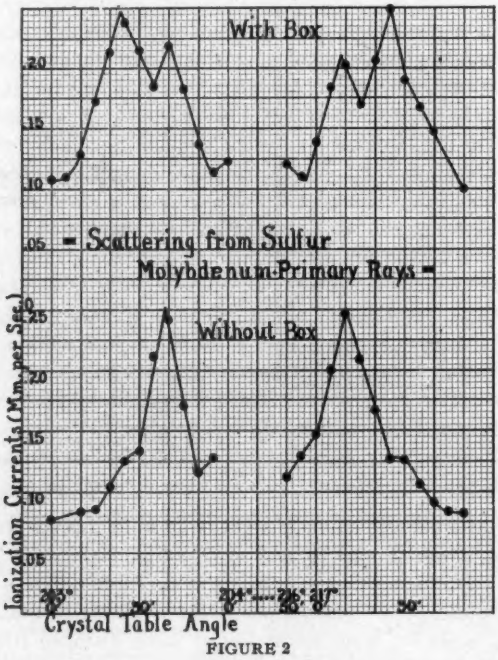

FIGURE 2

equaled 2 mm.) Consequently the resolving power is not as high as might well be obtained with apparatus especially designed for the investigation of these weak spectra. The resolving power, however, amply suffices to show the existence of radiation having both the wave-lengths of the primary rays and the wave-lengths somewhat longer. The actual wave-lengths calculated by the Bragg equation, $\lambda = 2d\sin\theta$, with $d = 3.028$ Ångström, amount to $\lambda = .711$ for the unmodified rays, and $\lambda = .738$ for the modified rays. The former is close to the weighted mean value of the wave-

lengths .708 and .714 of the two lines in the $K\alpha$ doublet of molybdenum. The wave-length shift in this case equals .027. The position of the modified peak corresponds very well with the positions of the shifted lines obtained in the experiments reported from other laboratories, l.c. The modified peaks also correspond in position very well with the peaks on our own experimental curves representing tertiary radiation from carbon and oxygen atoms due to primary rays in the K series of molybdenum, l.c. These results agree with those published in the preceding note.

The lower curves in our figure 2 represent an experiment performed after the wooden box had been removed and with no other change in the apparatus. It is at once apparent that the spectrum obtained without the box differs fundamentally from that obtained with it. The peaks representing rays with wave-lengths longer than the primary have disappeared practically completely. Small bulges on the long wave-length sides of the peaks that represent unmodified radiation still persist, but these are no larger than might be interpreted as due to tertiary radiation produced by the $K\beta$ line of molybdenum ejecting electrons from the K levels of the sulfur atoms and also to tertiary radiation produced by the $K\alpha$ doublet of molybdenum ejecting electrons from the L levels of the sulfur atoms.

Other experiments, similar to that described above, with the radiator placed in different positions inside of the box indicate that the relative intensities of the modified and unmodified peaks and the positions or wave-lengths of the *maximum* points of the modified peaks are very sensitive to slight changes in the experimental conditions. The complete elucidation of this point demands a long series of very careful experiments.

In these experiments, also, the tertiary radiation from the sulfur appeared as previously discovered and reported. This agrees with recent experimental results obtained in other laboratories. The intensity of the tertiary radiation with the box in place was quite small as compared with the truly scattered radiation.

The important thing for our purposes at present is that the peaks representing X-rays with shifted wave-lengths disappear when the wooden box is removed. They must, therefore, be due to a kind of box effect, and since they occupy the position in the spectrum corresponding to tertiary radiation from carbon and oxygen atoms, the presumption is that they are due to the tertiary radiation coming from the carbon and oxygen atoms composing the wooden walls of the box.

An experiment similar to that described above without the secondary radiator but with the box in place, gave substantially the same spectrum, except that its intensity was much weaker. This seems to prove that the spectrum obtained from the secondary radiator surrounded by the box was profoundly influenced by the reaction of the box on the radiator.

These experiments indicate that, if we look at X-radiation from the point of view of the emission of radiation quanta, having a certain amount of energy and momentum, we must assume that the momentum and energy are transferred not to single electrons but to atoms and groups of atoms, substantially as represented in a theory recently published[5] on the transfer of radiation momentum to matter in quanta.

[1] NATIONAL RESEARCH FELLOWS.

[2] THESE PROCEEDINGS, **9**, 413; **9**, 419 (1923); **10**, 41; **10**, 92; **10**, 148; **10**, 191 (1924).

[3] Compton and Woo, THESE PROCEEDINGS, **10**, 271 (1924); Ross, THESE PROCEEDINGS, July, 1924; Becker, THESE PROCEEDINGS, August, 1924.

[4] *J. Optical Soc. Am. & Rev. Sci. Insts.*, **8**, 681 (1924).

[5] THESE PROCEEDINGS, **9**, 158 (1923).

FURTHER EXPERIMENTS ON THE REFRACTION OF X-RAYS IN PYRITES

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This is a report of further experiments on the refraction of X-rays in pyrites. The previously reported experiments (*Proc. Nat. Acad. Sci.*, **10**, No. 2, Feb., 1924) were made with the characteristic K radiation from molybdenum, the present experiments were made with the K characteristic radiation from copper. The method used is that of the reflecting the rays from a crystal surface ground to an angle φ to the reflecting planes. This method is described in the previous report.

The refractive effect depends very much on the frequency of the incident radiation. In general, it increases as the square of the wave-length. The longer wave-length radiation was used to test the applicability to X-ray phenomena of the Lorentz dispersion formula:

$$\delta = \frac{e^2}{2\pi m}\left[\frac{n_1}{\nu^2 - \nu_1^2} + \frac{n_2}{\nu^2 - \nu_2^2} + etc\right],$$

where n_1, n_2, etc., are the number of electrons per unit volume having the resonance frequencies ν_1, ν_2, etc. In the above expression $\delta = 1 - \mu$ where μ is the index of refraction.

In the case of the refraction of the copper K characteristic in pyrites, the critical absorption frequencies of iron and sulphur are sufficiently close to the incident frequency to increase the refractive effect and to show a marked dispersive effect.

In order to compare the Lorentz dispersion formula with the experimental results it was expressed in the following form:

$$\delta = \frac{e^2N}{2\pi m}\left[\frac{K_{Fe}}{\nu^2 - \nu_{Fe}^2} + \frac{K_s}{\nu^2 - \nu_s^2} + \frac{(Z_{Fe} - K_{Fe}) + 2(Z_s - K_s)}{\nu^2}\right],$$

where N = the number of FeS_2 molecules per cc.

Z = the Atomic Number.

K_{Fe} and K_s = the number of electrons in K ring of each iron and each sulphur atom.

ν_{Fe} and ν_s are the K critical absorption frequencies of iron and sulphur.

The L M, etc., absorption frequencies are small compared to that of the incident radiation and may be neglected.

The experiments were carried out with an optically calibrated ionization spectrometer. The angles measured are angles of setting of the crystal. No vernier readings are made. All readings are on a calibrated tangent worm and hand-wheel.

The refractive bending of the beam of X-rays on passing through the crystal surface is greatly increased by grinding the surface to an angle to the planes. For example, in the case of copper $K\alpha$ radiation incident on a pyrites crystal, the bending is 12″ arc for case when crystal surface corresponds to crystal planes. When, however, the surface is ground to an angle of approximately 15° to the crystal planes, the bending of the beam is 210″ arc.

The experimental results are given in the last column of the table. This table also shows the calculated values of δ for $K = 0, K = 1, K = 2, K = 3$. That is, one, two or three electrons in the K ring of an atom. In every case the remaining electrons are assigned to the outer rings. If $K = 0$ the formula reduces to:

$$\delta = \frac{e^2}{2\pi m}\frac{n}{\nu^2},$$

which applies to the case of a cloud of electrons without resonance frequencies.

The values of the K critical absorption wave-lengths for iron and sulphur used in the calculations were 1.74 $A°$ and 5.023 $A°$, respectively.

RADIATION	λ	$\delta \times 10^6$ (CALCULATED)				$\delta \times 10^6$ OBSERVED
		K = 0	K = 1	K = 2	K = 3	
MO$K\alpha_1$	.7077	3.29		3.31		3.35 ± .20
MO$K\beta_1$	.6310	2.62		2.64		2.87 ± .20
Cu$K\alpha_1$	1.537	15.58	16.58	17.6	18.61	17.6 ± .5
Cu$K\beta_1$	1.389	12.69	13.12	13.53	13.95	13.2 ± .4

In the case of the Cu $K\alpha_1$ radiation the calculated values for various

values of K differ sufficiently to permit of experimental comparison. The agreement is best for two electrons in the K ring. If future experiments completely establish the validity of the Lorentz dispersion formula in its application to X-ray phenomena, then this furnishes a means of measuring the number of electrons in the K ring of atoms. It is hoped to extend the method to measurements of the number of electrons in the L ring as well.

These results, together with those previously reported on the refraction of X-rays in calcite (Davis and Hatley, *Proc. Am. Phys. Soc., Physic Rev.*, Feb. 1924), indicate that the Lorentz dispersion formula may be valid in the case of X-radiation. This is of great interest and importance from the point of view of the quantum theory. The Lorentz theory is based on the principle of resonance. Each term grows large as the frequency of the incident radiation approaches that of any group of electrons in the atom.

A quantum theory of dispersion must take account of the fact that at the critical absorption frequencies, the reactions of the electrons are similar to those of any other electro-mechanical system.

AN IMPROVED METHOD OF MEASURING THE SPECIFIC HEATS OF METALS AT HIGH TEMPERATURES

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Some time ago a method of using thermionic currents in the determination of the heat capacity of an incandescent filament was devised by one of us.[1] One strip of a double high frequency oscillograph carries the thermionic current emitted by the filament; the other strip, with a suitable resistance, is connected in series with the filament, or parallel with it. As the thermionic current changes by a considerable amount when the temperature of the filament changes slightly, the cyclic variations in temperature of a filament heated by an alternating current can easily be shown. In this case, the thermionic current is a direct current on which is superposed an alternating current whose frequency is twice the frequency of the heating current. Also the thermionic current reaches its maximum value after the heating current has passed through its maximum value. If the thermionic emission has been measured at a number of steady temperatures, it is possible to determine the amplitude of these cyclic variations in temperature, as well as the lag from a study of the photographic record made by the oscillograph.

Now, as Corbino[2] has shown there is a simple relation between such variations in temperature and the heat capacity of the filament.

Let c = mean heat capacity of the filament,
W = mean power supplied to it,
ω = 2π times the frequency of the heating current,
θ = amplitude of the variation above and below the mean temperature,
ϕ = angle of lag of temperature behind the power.

Then $c = -W \sin\phi/2\omega\theta$

Since the results of preliminary observations on a tungsten filament were published,[1] we have improved the experiment in certain respects, and have made a more careful determination of the specific heat of tungsten at temperatures between 2375° K. and 2475° K.

A straight tungsten filament of small diameter is mounted in the axis of a cylindrical glass tube. The heating current enters and leaves through copper leads, and in order to avoid errors introduced by the cooling effect of these leads, we have attached very fine tungsten wires a few centimeters from each end of the filament. These wires serve as "potential leads" and also carry the thermionic current back to the filament. A positively charged molybdenum cylinder surrounds the middle portion of the filament, while guard rings, negatively charged, prevent the escape of electrons from the ends of the filament. The thermionic current flows to the middle cylinder and passes through a d.c. milliammeter, 220 volt d.c. generator and one strip of the oscillograph. It divides into two equal parts at the middle of a high resistance which is connected across the potential leads.

The glass tube is heated in a furnace and pumped out until a high vacuum is obtained. After the filament has been aged sufficiently, a series of measurements of thermionic currents at various temperatures is made. In this part of the experiment a steady direct current is used to heat the filament, and the temperature is determined by means of an optical pyrometer.

Then a sinusoidal electromotive force is applied to the circuit, which otherwise remains unchanged. The mean power input, W, is calculated from the voltmeter and ammeter readings, a correction being made on account of the currents carried by the voltmeter and the high resistance shunt. The second strip of the oscillograph, shunted by a low non-inductive resistance, is placed in series with the filament. This arrangement seems more satisfactory than the earlier one in which this strip was in series with the voltmeter. The sensibility of the oscillograph is calculated for each plate by dividing one half the sum of the maximum and minimum deflections, for thermionic current, by the mean current. The maximum and minimum temperatures are found on the assumption that the instanta-

neous emission at any temperature is equal to the steady thermionic current observed when the filament is kept constantly at this same temperature. The amplitude of the temperature variation, θ, is one half the difference between the maximum and minimum temperatures.

The lag, ϕ, is found to be very nearly 90°, or in other words, the filament is at its mean temperature when the heating is zero, or a maximum. This result is in agreement with the findings of Corbino, Hunkel,[3] and others.

On account of the fact that the thermionic current is not a linear function of the temperature, its trace is not a true sine wave. It appears, however, that this does not affect the determination of the maximum and minimum temperatures.

The following table shows the mass and dimensions of the tungsten filament, and also the observations made for one determination of the specific heat:

Mass, between potential leads = 0.00248 gm.
Length between potential leads = 11.53 cm.
Diameter = 0.0038 cm.
Heating current (corrected) = 0.343 amp.
Potential difference = 27.1 volts
Mean power, W = 9.3 watts.
Average thermionic current = 0.050 amp.
Maximum deflection = 42.0 mm.
Minimum deflection = 26.0 mm.
Maximum temperature = 2491° K.
Minimum temperature = 2439° K.
θ = amplitude of temperature variation = −26° ϕ = lag = 89°
$\omega = 2\pi \times 60$
c = heat capacity of filament = 0.000114 cal./deg.
S = specific heat = 0.046 cal./gm. deg.

The mean of a number of determinations of the specific heat of tungsten at temperatures between 2375° K. and 2475° K. is 0.045 calories per gm. per degree. The atomic heat is therefore 8.3 calories per gram-atom. This agrees very well with the results obtained by Worthing,[4] and by Gaehr[5], using different methods.

A more extended account of this work will be published by one of us elsewhere.

[1] K. K. Smith and P. W. Bigler, *Physic Rev. Ithaca*, **19**, 268–270, 1922.
[2] O. M. Corbino, *Physik Zeits. Leipzig*, **11**, 413–417, 1910.
[3] H. Hunkel, *Ibid.*, **24**, 252–257, 1923.
[4] A. G. Worthing, *Physic. Rev. Ithaca*, **12**, 199–225, 1918.
[5] P. F. Gaehr, *Ibid.*, **12**, 396–423, 1918.

NOTE ON THE PRESSURE AND CURRENTS IN THE SUN'S ATMOSPHERE

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The large number of solar wave-lengths measured at Mount Wilson Observatory by grating spectrographs and by interferometers together with the iron-arc wave-lengths adopted at the Rome meeting of the International Astronomical Union furnish sufficient data to justify a reinvestigation of the question of pressure, in the sun's atmosphere.

If two groups of lines have pressure coefficients C_2, C_1 (per atmosphere), and if $\Delta\lambda_2$, $\Delta\lambda_1$ are changes in wave-length due to increase of pressure in passing from vacuum to sun, the pressure in atmospheres, in the sun's reversing layer is $(\Delta\lambda_2 - \Delta\lambda_1)/(C_2 - C_1)$.

The available coefficients of pressure are revised values from laboratory investigations not fully prepared for publication. By using sets of lines having approximately the same intensity, great differences of level with possibly unequal convection currents are avoided. By further limiting the constitution of the sets to lines in approximately the same spectral region, both Doppler and relativity effects become negligible. Eleven such sets were formed by selecting ten neighboring lines having large pressure coefficients and, closely surrounding and intermingled with these, an equal number of lines having smaller pressure coefficients. For six of the eleven sets the calculated pressure is positive, ranging from $+0.07$ to $+0.75$ atm. For five sets it is negative in sign, ranging from -1.00 to -0.09 atm. The weighted mean is $+0.13 \pm 0.06$ atm., which indicates a pressure that for most spectroscopic work is negligible in amount. Pressures of this order have also been found by Evershed,[1] Perot,[2] and Salet.[3]

Low pressures in the solar atmosphere have also been deduced from the theory of ionization. Thus Saha[4] estimates that the pressure at the upper level (5000 km.) of normal calcium is 10^{-4} atm. St. John,[5] from the increased intensity of enhanced lines over faculae, deduced a pressure of 10^{-2} atm. at medium levels. According to Russell[6] the increased intensity of Fraunhofer lines in passing from the general solar surface to spots is accounted for by a percentage of ionization which occurs at pressures not greater than 10^{-2} atm. Stewart[7] concludes that at the depth where the pressure is as great as 10^{-2} atm. the opacity of the ionized gas is sufficient to cut off the light from lower levels. Fowler and Milne,[8] on the assumption that Ca+ is in equilibrium under radiation pressure and gravitation, find a mean pressure of 10^{-13} atm. for the upper 5000 km of this vapor.

SUGGESTED SCHEME OF THE SUN'S ATMOSPHERE
DISTRIBUTION OF FACTORS ACCORDING TO LEVEL

COMPOSITION	CURRENTS SPOTS	CURRENTS DISK	PRESSURE	ROTATION 0° - 45°	HEIGHT
Ca + H + K LINES	3.8 Km, 2.0	0.5 Km	10^{-13} ATM	$\bar{\xi} = 15.^{\circ}4$ $\bar{v} = 2.18$ Km	14,000 Km
HYDROGEN Hα	3.0 Km			$\bar{\xi} = 14.^{\circ}65$ $\bar{v} = 2.09$ Km	
HYDROGEN Hβ	2.1 Km				8,000 Km
Ti^{+} λ3685	1.0 Km				6,000 Km
SODIUM D_1, D_2	0.4 Km				
NEUTRAL CALCIUM λ4227	0.1 Km		10^{-4} ATM	$\bar{\xi} = 14.^{\circ}44$ $\bar{v} = 2.06$ Km	5,000 Km
AL 15-20 Fe 15-40	0.0 Km	0.3 Km			1,500 Km
Fe 10	0.2 Km				1,000 Km
Fe 4	1.0 Km	0.0 Km	10^{-2} ATM	$\bar{\xi} = 13.^{\circ}84$	400 Km
Fe 00	2.0 Km	0.3 Km	10^{-1} ATM	$\bar{v} = 1.97$ Km	275 Km

Its uppermost limit shown by the H and K lines in the flash spectrum is, according to Mitchell,[9] 14,000 km.

In the study of solar and stellar atmospheres questions of pressure, levels and currents come necessarily into consideration. The accompanying chart is an attempt to sketch in tentative outlines the character and relative distribution of these factors in the sun's atmosphere. In the first column, under "composition," a few elements are arranged in the order of the magnitude and direction of the Evershed effect, indicated in the second column by the length and direction of the arrows.[10] The velocities represented in the third column refer to the undisturbed disk of the sun and appear to be the result of spectrographic integration over wide areas of the surface, at some points of which the vapors are rising and at others descending.[11] In the fourth column is a résumé of the pressure data discussed in the body of the paper. In the fifth column are given the mean angular and linear velocities from 0° to 45° latitude for Ca+ (St. John) for $H\alpha$, normal calcium, and low-level lines of iron and lanthanum (Adams). In the sixth column the relative heights of the factors are interpreted in kilometers by means of Mitchell's measures of flash spectra. In the case of weak or low-level lines it is to be remarked that the high temperature of the low-lying vapors weakens them as absorption lines, but the high temperature and the greater number of emitting centers in the line of sight at the limb strengthen them as bright lines in the flash, and that their relatively strong photographic action results in an overestimation of their height.

The chart represents our present interpretation of a wide range of observations. It is, however, not to be taken as definitive, but rather as suggestive of lines along which investigation may profitably be directed.

A fuller report will appear in *Mount Wilson Contribution*, No. 278.

[1] *Kodaikanal Bulletins*, No. 18, 1909; No. 36, 1913.
[2] *Comptes Rendus*, **172**, 1921 (578).
[3] *Ibid.*, **174**, 1922 (151).
[4] *Philosophical Mazagine*, **40**, 1920 (809).
[5] Contr. Jefferson Physical Laboratory, **15**, 1921.
[6] *Mt. Wilson Contr.*, No. 225; Astroph. J., **55**, 1922 (134).
[7] Physical Review, **22**, 1923 (324).
[8] London, Mon. Not. R. Astron. Soc., **83**, 1923 (403).
[9] Astroph. J., Chicago, **38**, 1913 (407).
[10] *Mt. Wilson Contr.*, No. 69; Astroph. J., **37**, 1913 (342).
[11] London, Mon. Not. R. Astron. Soc., **84**, 1923 (93).

CONVECTION CURRENTS IN STELLAR ATMOSPHERES

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In 1909–10 high-dispersion spectrograms of Sirius, Procyon and Arcturus were obtained by Adams and Babcock, the linear scale being 1.4 *A* per mm at λ 4300. The plates were measured by Adams, Miss Lasby, and Miss Ware. The principal result of the investigation at that time, which was the red displacement of enhanced lines relative to arc lines, was discussed by Adams.[1] In the meantime, studies of the relative behavior of solar and arc lines, the recognition of pole-effect, the classification of spectral lines into pressure and temperature groups, the theory of ionization, and the recognition of differences in level at which Fraunhofer lines have their origin have provided more adequate analytical methods of interpreting solar and stellar spectra, and a rediscussion of the original measures has accordingly been undertaken.

It has been shown that the displacements toward the red in the solar spectrum are larger for high-level than for low-level lines. This appears to be due to differences in radial velocity at different levels. At high levels the absorption produced by the cooler, downward-drifting vapors is the predominating influence in the spectrographic integration, while at low levels the ascending currents are chiefly effective in forming the lines. The great body of data yielded by the early spectrograms of high dispersion provides a means of testing whether similar conditions prevail in the atmospheres of stars.

Two groups of arc lines were formed, one of lines which, in the flash spectrum, according to Mitchell,[2] extend upward 600 km or more, and the other of lines whose heights are below 600 km; these are designated respectively, by the terms "high" and "low." The relative displacements of these groups are given in the last column of table I. For the stars, these differences represent the excess of the relative displacements of the same high-level lines in star and sun over those of the low-level lines in star and sun. Further, the stellar and solar wave-lengths of large groups of enhanced and arc lines were compared in the same way. The resulting differences, enhanced *minus* arc, are in the fifth column of table I. In the case of the sun, the displacements compared were those relative to laboratory values.

The striking features are, first, the close agreement of the differences between enhanced and arc lines with those between high- and low-level lines; and second, the decrease in these differences from Sirius through Procyon to Arcturus. Since the lines of ionized atoms are known to occur at greater heights in the atmosphere than those of neutral atoms, the greater

red-displacement of enhanced lines relative to arc lines can be attributed to the same cause as the differences between high- and low-level arc lines. In both cases the differences appear to represent the influence of a variation in convection currents with level.

TABLE I

RELATIVE DISPLACEMENT OF ENHANCED AND ARC LINES IN STELLAR SPECTRA COMPARED WITH DISPLACEMENTS OF ARC LINES OF HIGH AND LOW LEVEL

	NO. PLATES	NO. OF LINES ENHANCED	NO. OF LINES ARC	ENHANCED *minus* ARC	NO. OF LINES HIGH LEVEL	NO. OF LINES LOW LEVEL	HIGH *minus* LOW
Sirius	4	141	176	+0.016A	64	113	+0.015A
Procyon	4	107	278	+0.008	73	222	+0.006
Arcturus	14	316	1070	+0.001	80	666	+0.002
Sun		4(Ti)	13(Ti)	+0.004		Mean, Fe Lines	+0.003

The displacement of the stellar lines in table I, which are relative to those of the same lines in the solar spectrum, can be reduced to absolute values by combining them with the solar displacements relative to laboratory values. The results for the high- and low-level lines, are given in table II, together with the equivalent velocities in kilometers. The spectral types are added for comparison, as well as the temperatures according to Seares.[3]

TABLE II

RELATION OF DISPLACEMENT TO TEMPERATURE

	TYPE	TEMPERATURE	DISPLACEMENT HIGH LEVEL *minus* LOW LEVEL	
Sirius	A_2	8800°	+0.018A	+1.20 km/sec.
Procyon	F_5	6400	+0.010	+0.67
Arcturus	K_0	3900	+0.005	+0.34
Sun	G_0	5800	+0.003	+0.20

The displacements given in table II, interpreted as Doppler shifts, are in general agreement with the assumption that convection currents are the more pronounced the higher the temperature of the star. There is, possibly, also an influence depending on density.

The principal lines used in the determination of the radial velocities of the B-type stars are high-level lines of elements such as H, He, Mg, Si, O, and N. In the spectrum of Sirius the relative displacement of arc lines for moderate differences of level is 0.018 *A* at λ 4500, or 1.2 km, while for $H\gamma$ it is equivalent to 2.0 km. From these data it seems quite reasonable to conclude that at the still higher temperatures of the B-type stars the downward velocities may have a value of the order of 4 km, the amount of the *K*-term. These results strongly favor the hypothesis suggested several years ago by Campbell[4] that convection currents are a possible explanation of the *K*-term.

The pressures in the atmospheres of these stars may be referred directly to the pressure in the sun's atmosphere by determining the relative dis-

placements, star *minus* sun, for groups of lines whose pressure coefficients differ by determined amounts. Table III shows the results of a comparison of lines of classes *c* and *d* having large pressure-shifts with lines of classes *a* and *b* whose pressure-shifts are smaller.

TABLE III

RELATIVE DISPLACEMENT OF LINES WITH LARGE AND SMALL PRESSURE COEFFICIENTS

	NO. PLATES	NO. LINES	CLASS	DISPLACEMENT	PRESSURE (ATMOSPHERES)
Sirius	7	70	*c,d*	+0.001	+0.4 ± 1.1
		87	*a,b*		
Procyon	4	71	*c,d*	−0.003	−0.8 ± 0.9
		158	*a,b*		
Arcturus	11	175	*c,d*(*Mn*)	−0.002	−0.5 ± 0.8
		247	*a,b*		
Sun	—	—	—	——	+0.13 ± 0.06

The inference from these results is that the pressures in the atmospheres of these stars are low and of the same order as that in the sun. A similar investigation by Salet, based upon a much smaller number of lines, gave provisional values of about three atmospheres for Arcturus and one or two atmospheres for Procyon.[5]

A fuller account of this investigation will appear in *Mount Wilson Contribution*, No. 279.

[1] *Mt. Wilson Contr.*, No. 50; Astroph. J. **33**, 1911 (64).
[2] Astroph. J., **38**, 1913 (107).
[3] *Mt. Wilson Contr.*, No. 226; Astroph. J. **55**, 1922 (165).
[4] Lick Obs. Bull., No. 257, 1914.
[5] Astroph. J., **53**, 1921 (327).

A STATISTICAL DISCUSSION OF SETS OF PRECISE ASTRONOMICAL MEASUREMENTS, III; MASSES OF THE STARS

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Inasmuch as present-day conceptions in physics identify mass and energy through the relation $E = mc^2$, and both mass and energy are found in their greatest concentrations only in the stars, there is physical and astronomical interest in the study of the relation of stellar mass to radiant energy. Neither the mass nor the radiant energy has been directly measured for many stars; both have for the most part been estimated by various devices of a statistical and hypothetical nature aiming chiefly at reasonableness of astrophysical assumption and self-consistency of statistical

evidence. For the most part this is at present the only available approach, but the ideal is always present of checking the results against directly measured cases so far as may be.

Let us examine the statistical evidence regarding measured masses. Writing in 1914 Eddington (*Stellar Movements*) stated that the masses of only 8 systems were known at all well; of the sun and of 7 binaries with known parallaxes. The figures expressed in the solar mass as unit are in order of magnitude 0.7, 1.0, 1.0, 1.3, 1.8, 1.9, 2.5, 3.4. The arithmetic mean is 1.7 ± 0.2. As no masses may be negative and as there is no restriction, except through disintegration by internal light-pressure or dynamical fission, upon the upper side, it is natural to discuss the distribution not of the mass itself but of its logarithm. The 8 results are (base 10) 9.845, 0.000, 0.000, 0.114, 0.255, 0.279, 0.398, 0.532. The mean is $0.178 \pm .05$, the standard deviation of the individual data being $\sigma = 0.21$ and the probable error $\rho = 2\sigma/3 = 0.14$. We may write $\sigma = 0.21 \pm 0.03_5$. The geometric mean mass is therefore 1.5 and coincides with the median. If the distribution may be considered normal, as certainly cannot be maintained for so few items even though the tests for normality appear to be adequately satisfied, a departure of $9\rho = 1.26$ in the logarithm from its mean 0.18 could not occur; because there is only about one chance in a billion and a half that $\log m$ should exceed 1.44 and a like chance that it should be less than 8.92, and there are probably less than one billion stars nearer than 2000 parsecs with the certainty that no such number are possible of registration on the best photographic plate. There are at least the 3 known binaries of which the mass exceeds $10^{1.44} = 27.5$. It is clear, therefore, that a normal distribution as determined from these data is not in fact astronomically existent.

Writing in 1923 Hertzsprung (*Bull. Astr. Inst. Neth.*, **2**, 15, No. 43, 1923) gives the following 15 systems as having reasonably well determined masses.

$7.50 = 4.18 + 3.32$	$1.13 = 0.89 + 0.24$
$4.72 = 2.38 + 2.34$	$1.09 = 0.58 + 0.51$
$3.41 = 2.45 + 0.96$	1.00 (sun)
$2.57 = 1.85 + 0.72$	$0.93 = 0.60 + 0.33$
$2.11 = 1.14 + 0.97$	$0.88 = 0.44 + 0.44$
$1.82 = 0.96 + 0.86$	$0.43 = 0.30 + 0.14$
$1.60 = 1.12 + 0.48$	$0.41 = 0.21 + 0.20$
$1.50 = 1.13 + 0.37$	

The arithmetic mean for the systems is $1.97 \pm .28$; the median is still 1.5; the real interest, however, lies in the distribution of the logarithms. The mean is 0.18 ± 0.06; the standard deviation $\sigma = 0.34 \pm .04$. The mean deviation $\vartheta = 0.27$ and the test $\sigma = 1.25\vartheta$ for normality is perfect. Furthermore the study of the distribution of the logarithms about their mean with regard to the size of the probable error $\rho = 0.23$ for the individ-

ual datum gives added evidence of the satisfactory normality of the frequency function. The geometric mean mass is 1.5 again. On comparing the result with the calculation for the previous 8, it appears that the mean has not moved, that its probable error has increased 20% despite the larger number of cases, and that the probable error of the individual datum has increased 60%, thus indicating a decidedly greater dispersion about the mean. In fact it is difficult even considering the small size of the samples (8 and 15) to reconcile the relative magnitudes of the standard deviation and their probable errors; the difficulty would have been even greater if the additional 7 had been treated as a second sample. The fact is that the two sets probably do not belong to the same statistical universe—one at least of the samples is not fair. Of course it is not easy to see how a sample of 8 or even 15 can be fair; but owing to paucity of material statisticians even in astronomy often operate with very small samples. The illustration shows pointedly that generalizations, i.e., statistical inferences from a small sample, are likely to be wide of the mark. Again, if charmed with the excellence of the normal distribution of the 15 cases, we should try to determine the probability that a binary should have log m as great as $0.18 + 9\rho = 2.21$ we should find not one chance in a billion and a half for a star of mass 162, that is, an impossibility by a wide margin—and yet we have the Plaskett binary already observed with just about that mass.† It might be argued that owing to the great errors in the parallaxes the original data are not good, and certainly they are not good to the three figures Hertzsprung gives. Let it be admitted that the figures are inaccurate by 30% which means a variation of about 0.12 in the individual logarithms. If there is nothing systematic in the errors of 0.12 we should expect an error in the mean of about 0.03 which is but half of the probable error found. So far therefore as accidental errors go, the mean is reasonably well established though its error might increase. With systematic errors, such as over-evaluation of parallaxes the matter is different; the mean might rise considerably while its error might not.

A similar discussion may be given for the 29 stars on Hertzsprung's list when each component of a binary is counted separately. Then it is found that the median mass is 0.86 and the arithmetic mean is 1.07 ± 0.12. The distribution is skew. If, however, we distribute the logarithm we find the median as 9.934 and the mean as $9.984 \pm .044$ corresponding to a geometric mean mass of 0.965 determined to a factor of 1/9. The standard deviation is $\sigma = 0.357 \pm 0.030$. Again the mean deviation ϑ checks perfectly with σ and the examination of the 29 logarithms with respect to distribution indicates the same high degree of normality as in the case of 15. It is interesting that the dispersion σ about the mean is not significantly greater for the 29 stars than for the 15 systems. The mean is better determined statistically than before in just the ratio ex-

pected from the increase in the number of data. The probable error of the individual deviation is 0.24. If again the excellence of fit of the distribution should tempt us to pass from statistical description to statistical inference, we should find fewer than 1 binary in 2700 with the mass of either component so great as 15; fewer than 1 in 40,000 for 26; fewer than 1 in 850,000 for 46; and fewer than 1 in 30 million for 63—yet it is probable that both the components of the Plaskett binary exceed the last figure. It is perhaps fortunate that the stars of large mass thus far discovered are so few, because there are fairly convincing theories that set rather low limits to the possible mass of any one star; it is, however, difficult in making speculative theories to avoid being prejudiced by preconceptions of what should be found and these preconceptions may be unconsciously influenced by what is known with respect to an even very limited range of data.

As Jeans well says (*Cosmogony and Stellar Dynamics*, p. 288) "The time for arriving at conclusions in cosmogony has not yet come"; but we may sum up this discussion as follows: So far as we have reasonably good data the distribution of the logarithm of the mass of the stars, alike when we take the binaries as systems and when we resolve them into their components is extraordinarily close to normal, much closer than could be expected for so meager data. The geometric mean mass is well determined in both cases and the additional dispersion that would probably be introduced by accidental errors in the values given by Hertzsprung would not be great nor much effect the mean. In making observations and detecting binaries one does not operate by chance in the sidereal universe as a whole but is necessarily largely limited by various methods of selection such as large proper motion, high apparent magnitude, large parallax, remarkable spectral class, etc., and it is therefore not to be expected that a small group of stars thus selected could be a fair sample of the stars at large. It is, therefore, all the more remarkable that the distributions of the samples should be so regular as they are and thus give a strong impression of randomness or fairness in the samples. It should be observed, however, that the three systems Y Cygni ($31.9 = 16.6 + 15.3$), V Puppis (33), Plaskett ($160 = 87 + 73$) do not in any way fit into the distribution above found; that if they be included the logarithmic mean for systems jumps from 0.18 ± 0.06 to 0.44 ± 0.13 (corresponding to a geometric mean mass of 2.7) and the standard deviation from 0.34 ± 0.04 to 0.84 ± 0.10; and with these inclusions not only has there been a complete disturbance of the statistical constants but the distribution has lost all its appearance of being seductively normal. If, on the other hand, we keep to the original and apparently stable distribution the discovery of the first two of these systems can be at least partly explained somewhat contradictorily by the very principles of selection which must operate (we should expect to find not the normal but the abnormal cases); but the

existence of a mass so great as 160 (or even the minimum assigned value 137) appears to be statistically impossible if we have regard to even the largest reasonable estimate of the number of binaries accessible to observation irrespective of parallax. It will require a good many more cases to bring back the previous appearance of statistical stability.

† While this article was in press Ludendorff (*Sitz. Preuss. Akad. Wiss.*, 1924, VIII–IX, p. 67) has announced minimum masses of 260 and 54 for the components of υ Sagittarii.

THE IONIZATION POTENTIALS OF HYDROGEN AS INTERPRETED BY POSITIVE RAY ANALYSIS

By T. R. Hogness and E. G. Lunn

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Communicated July 17, 1924

Up to the time that one of the authors first began this investigation, the interpretation of the data on ionization potentials of polyatomic gases had been almost wholly a matter of conjecture, since no attempts had been made to resolve the group of ions, produced by impact electrons of various velocities, into their constituent parts. It was with the object of interpreting these data that the present investigation was started, and, while the work was in progress, several announcements[1] of experimentations similar to that conducted by the authors have appeared.

After the apparatus had been perfected, hydrogen was the first gas to be investigated and, although the investigation of this gas has not been completed, it seemed that a preliminary note on the apparatus, together with the results so far obtained, would be of interest to those engaged in ionization potential and allied work.

General Description of Method.—The gas is ionized by impact electrons emitted by a hot tungsten filament, and, by means of an electric field the positive ions formed are drawn through a narrow slit into a magnetic field, where they are resolved into constituents of different ratios of charge to mass by a method very similar to that employed by Dempster[2] in his positive ray analysis. The ionization potential necessary to produce each ion is determined by gradually reducing the potential applied to the impact electrons until no trace of the particular ion can be detected.

Detailed Description of the Apparatus and the Method Employed.—After several trials the apparatus finally adopted is the one shown in Fig. 1. The gas enters the apparatus at A, passing into the ionization chamber H, through the slit at B, into the magnetic chamber C, through D, and im-

mediately into a liquid air trap and mercury diffusion pump of the type made by Mr. W. C. Cummings of this laboratory. The slit at B is 3 mm. long, and for these experiments was 0.5 mm. wide. With this slit opening, a fifteen-fold difference of pressure could be maintained between the chambers H and C.

The electrons are emitted by the tungsten filament E, accelerated toward the gauze G, and pass into the ionization chamber H. The potential between E and F is kept constant and, whenever desirable, the potential between F and G is varied. While varying the applied potential, a more constant stream of electrons should be obtained, with the extra gauze G, than with only one gauze; thereby causing a sharper break in the ioniza-

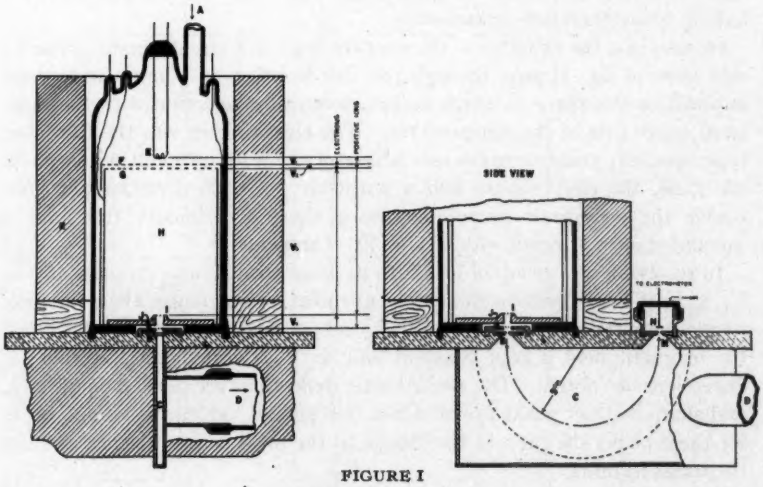

FIGURE I

tion potential curves. The platinum gauzes F and G are mounted on copper cylinders, to give them support and to insure equipotential walls within the ionization chamber H. All metallic parts of the apparatus are insulated from each other by glass cylinders or plates. The distance from the filament to the gauze F is about 5 mm., and the distance between the two gauzes F and G is 3 mm.

The positive ions are produced in the chamber H and are accelerated toward the small gauze I by a small field V_3, then accelerated again toward the gauze J by a large field V_4, and pass through the slit B. The gauze J and the slit B are at the same potential. The gauzes I and J are mounted on discs of the shape shown in the drawing. These discs and the knife edges of the slit are made of iron to act as magnetic shielding against the stray field of the magnet, and they are copper-plated to minimize the

desorption of the foreign gases held on the surface of the iron. The ionization chamber is 10 cm. long. This length allows a lower pressure in the chamber H, with a consequent lower pressure in the chamber C. No effects due to the stray magnetic field in H were observed since the whole glass portion of the apparatus is surrounded by a heavy iron casting K, which acts as a magnetic shield. The plate L and the walls of the chamber C were constructed of rolled brass plate. The chamber C has an internal width of 3 mm. The glass portion of the apparatus, together with the line and the McLeod gauge, were constructed entirely of Pyrex glass, and the glass portion of the apparatus was sealed to the plate L by means of de Khotinsky cement. The filament was run at so low a temperature and was so far removed from the glass walls that these never became warm; baking being therefore unnecessary.

On entering the chamber C the ions are bent in a circular path, (refer to side view of fig. 1) pass through the slit M which is 1 mm. wide, and impinge on the plate N which makes electrical connection with the insulated quadrants of the electrometer. The electrometer was the Compton type, specially constructed in this laboratory. With a 75 volt potential on the vane, the electrometer had a sensitivity of 6500 divisions per volt. Under the conditions prevailing during these experiments, this gave a calculated current sensitivity of 1×10^{-16} amperes.

In resolving any group of ions into its constituent charge-to-mass ratios, $V_1 + V_2$ is kept constant at some convenient value greater than the ionization potential, usually 30 volts. V_3 is kept constant at 2.0 to 4.5 volts, the magnetic field is kept constant and, as V_4 is varied, the electrometer deflections are noted. The electrometer deflections are plotted against V_4, and sharp distinct peaks are obtained (see fig. 2). From the values of V_4 for these peaks the ratio of the charge to the mass can be calculated from the usual formula,

$$e/m = 2V_4/H^2r^2.$$

Here r is the radius of curvature of the rays in the magnetic chamber, and in this work was 5 cm. V_3, V_4, and the current through the field coils are equipped with double-throw reversing switches, so that negative as well as positive ions could be studied.

The magnetic field was roughly calibrated by analyzing the positive rays of air. The peak representing the greater value of the charge to the mass was taken to be N^+, and the peaks for O^+, H_2O^+, $N_2{}^+O_2{}^-$ and a very small peak for A^+ appeared in their respective places. The relative intensities of these ions were also of the right order of magnitude; the sum of the maximum intensities of the N^+ and $N_2{}^+$ was about four times the sum of those for O^+ and $O_2{}^+$. A rough value of the field strength could be calculated from the above formula, and from this the positions of the peaks

for the ions of hydrogen determined. These ions invariably appeared in their calculated positions. An oxide-coated platinum filament was used in the calibration.

The Postive Rays of Hydrogen.—The hydrogen was prepard by electrolysis of a solution of sodium hydroxide and was dried over phosphorous pentoxide. The vacuum line and apparatus were washed out repeatedly to insure against contamination by residual gases. With a pressure of 1×10^{-4} mm. in the apparatus, it was possible to detect foreign gases to the extent of one part in a thousand, and in all of the later runs no trace of

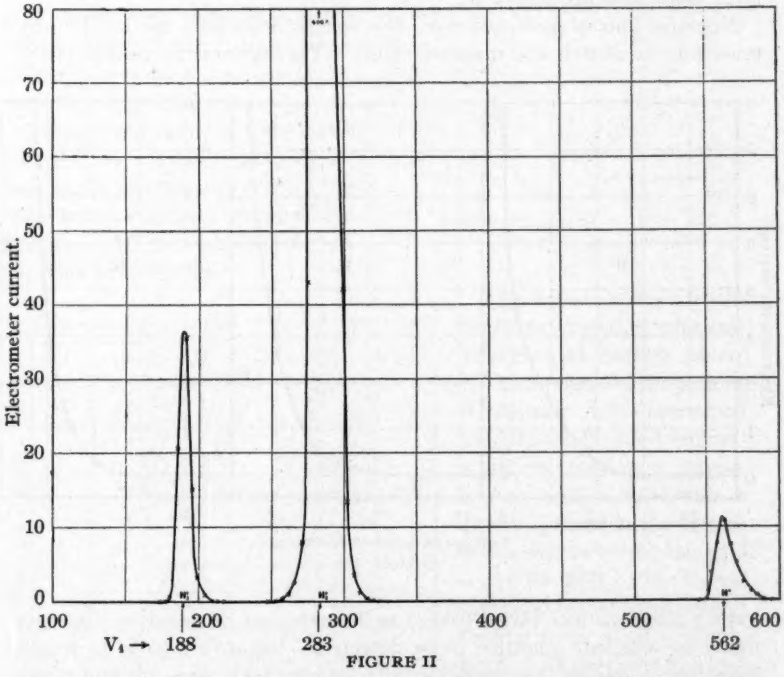

FIGURE II

foreign gas could be observed. The hydrogen was stored in a two-liter reservoir and was allowed to enter the vacuum line through a small capillary tube. With a pressure of one atmosphere in the reservoir, the pressure of the gas in the apparatus could be maintained indefinitely at 30×10^{-4} mm. To vary the pressure of the gas in the apparatus, the pressure of the gas in the reservoir was varied accordingly.

Three positive ions of hydrogen, H^+, H_2^+, and H_3^+ were found. In figure 2 a typical run is plotted with V_4 as abscissæ and the electrometer readings in centimeter scale divisions per 10 seconds as ordinates. $m/e = 1$ cor-

responds to the peak at 562 volts, $m/e = 2$ at 283 volts, and $m/e = 3$ at 188 volts. It will be observed that the intensity of the H_2^+ is very much greater than either the H_3^+ or the H^+. The pressure of the hydrogen at which this particular run was made, was 30×10^{-4} mm. With diminishing pressure H_2^+ continues to predominate, but the ratio of H_3^+ to H^+ becomes smaller. The relative intensities were often difficult to reproduce. To obtain appreciable deflections for either H_3^+ or H^+, the deflections for H_2^+ were so great that large errors were introduced in the stop-watch readings. A discussion of the relative intensities of the ions is postponed until more data are available.

Negative ions of hydrogen were also sought, especially the ion H^-, by reversing the electric and magnetic fields. The accelerating voltage of the

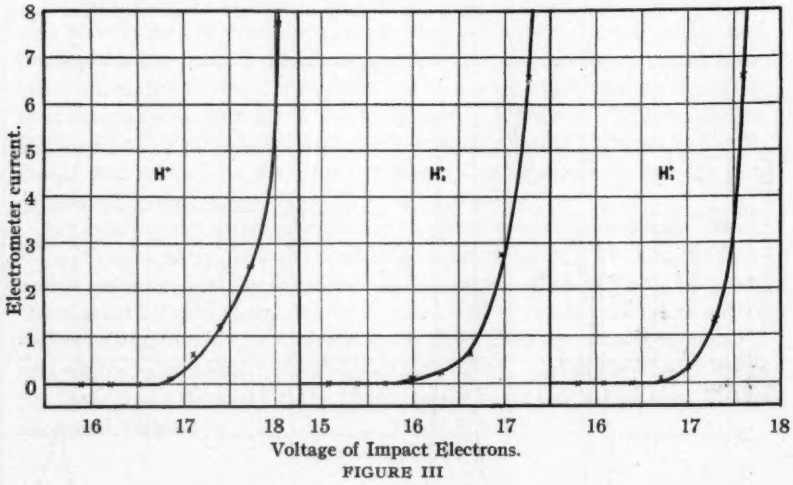

FIGURE III

impact electrons was varied from 2 to 30 volts, but no negative ions were found in sufficient quantity to be detected. Negative ions of hydrogen have been found by Thomson but at relatively high pressures and under other conditions that indicate that their formation is an effect secondary to ionization. Under the conditions prevailing during these experiments, it would hardly be expected that these should appear.

Ionization Potentials.—In determining the ionization potentials of any one of the ions, the filament was raised to a sufficient temperature to give a large electrometer deflection, such as shown for H_2^+ in figure 2, V_4 was set to give the maximum intensity for the ion, and the voltage of the impact electrons was diminished in steps of 0.3 volt until no electrometer deflection was detectable. The electrometer deflections were plotted against the

accelerating voltage of the impact electrons (fig. 3), and the ionization potential was taken to be that point at which the curve cut the zero ordinate. As a further check the accelerating voltage of the electrons was maintained at a value just below and above the ionization potential determined in this way, and the electrometer deflections were noted over a period of ten to fifteen minutes. In all of these checks the electrometer showed no deflections below the critical potential, while above this critical potential a very definite deflection was obtained.

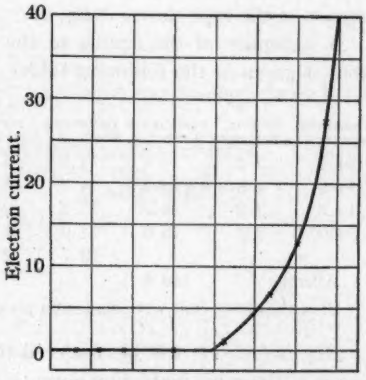

FIGURE IV

Figure 3 shows one of these curves for each ion. The accelerating voltage of the electrons $(V_1 + V_2)$ plus their initial velocities was plotted as abscissæ and the electrometer deflections as ordinates.

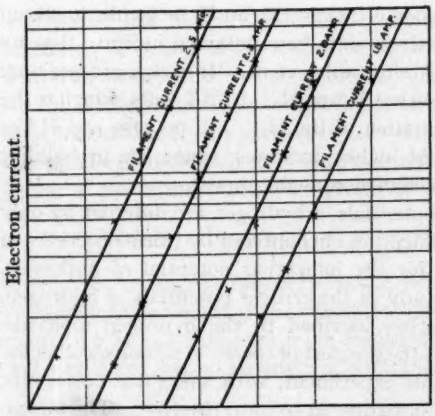

FIGURE V

The initial velocity of the electrons, together with any difference of contact potential and potential drop along the filament, was determined for different filament temperatures by placing a galvanometer with a sensitivity of 2500 megohms in the electrical circuit between the grid G and the grid I. As V_3 was increased the electron current flowing from G to I diminished. At the point at which no electrons penetrated to the plate I the correction was taken to be $V_3 - (V_1 + V_2)$. This value was best taken from a plot of the electron current against $V_3 - (V_1 + V_2)$. A typical curve of this kind is shown in figure 4. The correction was taken to be the point at which the curve cut the zero ordinate. The upper portion of the curve was logarithmic so that by plotting the electron current on a logarithmic scale a straight line

was obtained (fig. 5). Although the initial velocities could not be obtained from this plot, the differences of the velocities at different temperatures could. This afforded a check on the initial velocities selected from the curves of the type shown in fig. 4.

A summary of the results of the ionization potentials of the different ions is given in the following table.

H^+			H_3^+			H_3^+		
OBSERVED VOLTAGE	INITIAL CORRECTION	IONIZATION POTENTIAL	OBSERVED VOLTAGE	INITIAL CORRECTION	IONIZATION POTENTIAL	OBSERVED VOLTAGE	INITIAL CORRECTION	IONIZATION POTENTIAL
14.4	2.2	16.6	14.0	1.4	15.4	14.3	2.2	16.5
14.9	1.6	16.5	14.2	1.4	15.6	14.7	2.1	16.8
14.5	2.2	16.7	13.5	2.2	15.7	14.6	1.9	16.5
14.0	2.6	16.6	14.9	1.0	15.9	14.7	2.2	16.9
			13.7	2.2	15.9	14.0	2.6	16.6
Average		16.6			15.7			16.7

All of the above runs were made at a pressure of 30×10^{-4} mm.

Discussion.—It will be observed that, within the limits of error, H_3^+ appears at the same ionization potential as H^+. Since, as has been stated, the amount of H_3^+ increases with respect to H^+ with increase of pressure, it is natural to assume that H_3^+ is formed by the combination of H^+ with the neutral H_2 molecule, a spontaneous process which occurs with the evolution of energy. From energy considerations, it is possible that an H_2^+, sufficiently accelerated, could, upon collision with an H_2 molecule, break up to form either H^+ or H_3^+, yet the results here obtained indicate that no such reaction takes place to any detectable extent. If such a process took place we should expect to find either H^+ or H_3^+ at 15.7 volts, which is the ionization potential for the formation of the H_2^+. In no case was H^+ or H_3^+ found below 16.5 volts. At higher pressures, where the probability of collision is greater, such a phenomenon might appear.

The fact that the ionization potentials of hydrogen are different by only 0.9 volt may account for the difficulties encountered by other observers in arriving at concordant values for the ionization potential of hydrogen. Olson and Glockler[3] in their study of the critical potentials of hydrogen, found three potentials which they ascribed to the hydrogen molecule. It is exceedingly interesting that the greatest of these, 15.62 volts, coincides, within the limits of error of this experiment, with the value 15.7 volts, which represents the ionization of hydrogen to form the H_2^+. The ionization potential to form H^+, 16.6 volts, is also in agreement with their value of 16.68. We may therefore assume that the interpretation of the data of these observers is the correct one. Since the method which they employed is adapted to greater refinements than can be obtained with the apparatus used in this experiment, their values are undoubtedly the more correct ones.

These results show conclusively that there are two distinct primary processes taking place:

(1) $H_2 = H + H^+ + E^-$; 16.6 volts,
(2) $H_2 = H_2^+ + E^-$; 15.7 volts,

and one secondary process,

$H^+ + H_2 = H_3^+$.

Further discussion will be postponed until more experiments have been made.

[1] Kallmann and Knipping, *Naturwis*, **10**, 1014 (1922), preliminary notice.
H. D. Smyth, *Proc. Roy. Soc.*, **104**, 121 (1923); **105**, 116 (1924).
[2] A. J. Demptster, *Physic Rev.*, 11, 316 (1918). 11, 316 (1918)
[3] Olson and Glockler, *Proc. Nat. Acad. Sci.*, **9**, 122 (1923).

THE POLARIZING ANGLE FOR X-RAYS SCATTERED BY PARAFFIN

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Jauncey[1] has recently described a corpuscular quantum theory of the scattering of polarized X-rays, in which a formula for the intensity of the scattering in any direction ϕ is derived. From this formula an expression for the linear scattering coefficient per unit solid angle in any direction ϕ may be obtained. In particular this linear scattering coefficient for the case where plane polarized X-rays are scattered in the plane of the electric vector of the primary X-rays is given by

$$s = (Npd/W)\,(e^4/m^2c^4)\{(1+\alpha)\cos\phi - \alpha\}^2/(1+\alpha\,\mathrm{vers}\phi)^5, \quad (1)$$

where N is Avogadro's number, d is the density of the scattering substance, W its molecular weight, p the number of electrons per molecule, and α is the quantity $h/mc\lambda$, λ being the wave-length of the primary X-rays. It is seen that s becomes zero at an angle ϕ_p such that

$$\cos\phi_p = \alpha/(1+\alpha) \quad (2)$$

ϕ_p may be called the polarizing angle of scattering since when unpolarized primary X-rays are scattered at this angle the scattered X-rays are completely plane polarized. The present work was undertaken to test the truth of Eq. (2).

X-rays from a Coolidge X-ray tube with a tungsten target were directed on to a slab of paraffin A. By a suitable arrangement of apertures

in lead screens the X-rays scattered by the slab A in a direction 88° with the primary X-rays were allowed to fall upon a second paraffin slab B. The scattering in various directions from this second slab B was measured by means of an ionization chamber filled with methyl iodide and a Compton electrometer operating at a sensitivity of about 5000 mm. per volt. The plane of rotation of the ionization chamber was such as to contain the electric vector of the polarized X-rays coming from the paraffin slab A. The arrangement of the apparatus as far as the scattering from the block B is concerned is the same as that described by Jauncey and May[2] in their experiment on the scattering of X-rays by rock salt. The slab B was mounted on the axis of the Bragg spectrometer and Crowther's method[3] of obtaining the experimental value of the scattering coefficient s was used as described in the paper of Jauncey and May.[2] This method gives

$$s = I_\phi R^2 \cos\theta / SIt, \qquad (3)$$

where I_ϕ is the intensity of the X-rays entering the ionization chamber when it is set an angle ϕ with the primary beam, R is the distance of the chamber window from the slab B, S is the area of this window, t the thickness of the slab B, θ the angle between the normal to the slab and the primary beam, and I the intensity of the primary X-rays penetrating through the slab B. In Crowther's method $\theta = \phi/2$. In the present experiment there were of course no Laue spots as in the experiment of Jauncey and May and all readings were taken at $\theta = \phi/2$.

Experiment with Hard X-Rays.—The X-ray tube was operated at 150 kv. peak and 7 ma. In addition the primary X-rays falling on the slab B were

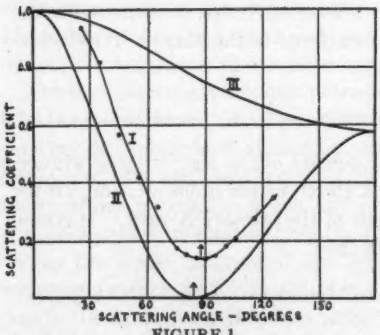

FIGURE 1

Variation of Scattering Coefficient with Angle. Hard X-Rays.

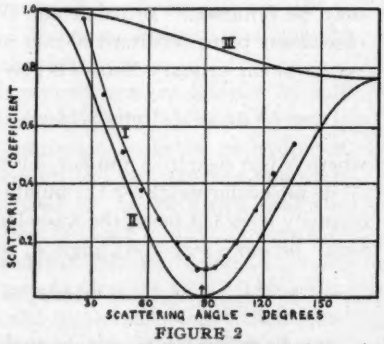

FIGURE 2

Variation of Scattering Coefficient with Angle. Soft X-Rays.

hardened by passing through 2 mm. of aluminum. The effective wavelength of the X-rays which had passed both through this aluminum and through the slab B was obtained by finding the thickness of aluminum

which reduced the intensity of these residual rays to one half. The mass absorption coefficient was then calculated and the effective wave-length determined from Hewlett's values[4] of the mass absorption coefficient for different wave-lengths. The effective wave-length was found to be 0.25 A. The thickness of the slab B was 1.61 cm. The experimental values of s are shown by the black circles in figure 1, curve I being the experimental curve. Curve II is Jauncey's theoretical curve for $\lambda = 0.25\ A$, or $\alpha = 0.096$, obtained from Eq. (1). The experimental curve has a minimum at an angle about 87° 30′, which is on the forward side of 90°. On Thomson's theory[5] this minimum should come at 90°. Jauncey's theoretical curve has a minimum at 84° 47′ as given by Eq. (2). The ordinates of the experimental and theoretical curves are on the same scale.

Experiment with Soft X-Rays.—The X-ray tube was operated at 70 kv. and 13 ma., the hardening slab of aluminum being removed. The effective wave-length was 0.54 A and the thickness of the slab B was 1.093 cm; otherwise the conditions were the same as when hard X-rays were used. The experimental values of s are shown by the black circles in figure 2, curve I being the experimental curve. Curve II is Jauncey's theoretical curve for $\lambda = 0.54\ A$, or $\alpha = 0.044$. The experimental curve has a minimum at an angle which cannot be distinguished from 90°, while on Jauncey's theory the minimum should be at 87° 30′.

It is seen that the minimum is shifted to the forward side of 90° when hard X-rays are used. The interpretation of these experimental results is somewhat uncertain since both Thomson's and Jauncey's theories require that when polarized rays are scattered at the polarizing angle s should be zero, whereas the ordinate of each of the experimental curves of figures 1 and 2 is not zero at the minimum. This finite value of the minimum ordinate is due to (*a*) lack of complete polarization of the primary beam, (*b*) angular width of the primary and scattered beams, and (*c*) multiple scattering in the slab B. Now if Thomson's theory is correct the effect of (*a*) and (*b*) will be to give a minimum with a finite value at 90°; while if Jauncey's theory is correct the effect of (*b*) will be to shift the minimum to an angle on the backward side of the polarizing angle.

Next let us consider the effect of (*a*) on Jauncey's theory. If I_0 is the total intensity of the primary X-rays and these are partially polarized, I_0 may be considered as made up of I_1 polarized with the electric vector in the plane of scattering and I_2 polarized with the electric vector perpendicular to this plane. The scattering coefficient s for I_1 will be given by Eq. (1), but that for I_2 will be given by (see Jauncey's paper[1])

$$s = (Npd/W)\,(e^4/m^2c^4)/(1 + \alpha\,vers\phi)^3 \qquad (4)$$

The theoretical value of s will be the weighted mean of Eqs. (1) and (4). Curves III in figures 1 and 2 give the values of s, Eq. (4), for $\alpha = 0.096$ and

$\alpha = 0.044$ respectively. If I_2 is not zero the mean theoretical curve will have its minimum shifted to the backward side of the polarizing angle.

The effect of (*c*) involves a difficult mathematical problem which the writers have been unable to solve. However an experimental solution can be found by obtaining scattering curves for different thicknesses of slab *B* for the same wave-length and extrapolating to zero thickness. The intensities are so small however that this will not be possible until a more powerful X-ray tube is obtained. The writers however can see no *a priori* reason why the multiple scattering should shift the minimum to the forward side of 90°. From the geometry of the set-up the angular width of the rays entering the ionization chamber window when the chamber was set at 90° could be calculated, and the extreme width was found to be about 24°. This width due to (*b*) would give an ordinate of the value 0.017 at the mimimum in the units of the curves in figures 1 and 2. It seems then that most of the value of the minimum ordinate is due to (*a*) and (*c*).

In conclusion it is seen that the effects of (*a*) and (*b*) on both Thomson's and Jauncey's theories would not cause the minimum to shift to the forward side of 90°. Therefore seeing that the experimental minimum is shifted to the forward side of 90° for short wave-lengths it seems that a shift of the polarizing angle in qualitative agreement with Eq. (2) has been observed, the only uncertainty being the effect due to multiple scattering. Another possible reason why the experimental minimum should come at an angle between the value given by Eq. (2) and 90° is that Compton[6] finds both modified and unmodified scattered X-rays in his experiments on the change of wave-length. The unmodified rays probably have their polarizing angle at 90°. Hewlett[7] and Jauncey[8] have observed a minimum at angles on the backward side of 90° when unpolarized X-rays are scattered. This is in accordance with Jauncey's theory since on giving equal weights to curves II and III and averaging we obtain a curve with a minimum on the backward side of 90°.

[1] G. E. M. Jauncey, *Physic Rev., Ithaca*, **23**, 313 (1924).
[2] Jauncey and May, *Ibid.*, **23**, 128 (1924).
[3] J. A. Crowther, *Proc. Roy. Soc.*, **86**, 478 (1912).
[4] C. W. Hewlett, *Physic Rev.*, **17**, 284 (1921).
[5] J. J. Thomson, *Conduction of Electricity through Gases*, p. 325.
[6] A. H. Compton, *Physic Rev.*, **22**, 409 (1923).
[7] C. W. Hewlett, *Ibid.*, **20**, 688 (1922).
[8] G. E. M. Jauncey, *Ibid.*, **20**, 405 (1922).

THE COMBINATION OF HYDROGEN AND OXYGEN IN THE PRESENCE OF ACTIVATED MERCURY

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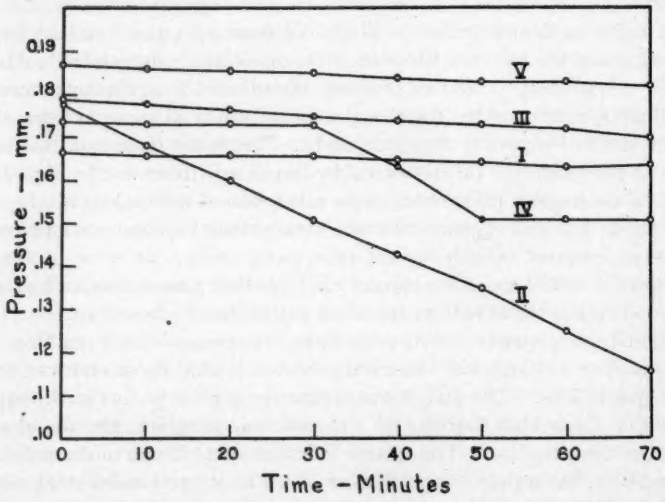
In order to demonstrate the ability of mercury atoms, excited by the absorption of the mercury line 2537 Å., to dissociate hydrogen by "collisions of the second kind," Cario and Franck[1] illuminated with a quartz mercury-arc lamp a mixture of hydrogen and mercury vapor in a quartz tube which also contained copper or tungsten oxide. The water formed in the reduction of the oxide by the activated hydrogen was removed by liquid air, and the consequent pressure decrease was followed with a MacLeod gauge. The writer has had occasion to make some similar experiments employing, however, gaseous oxygen instead of a metal oxide. In view of the importance of collisions of the second kind in other phenomena as well as in photo-chemical sensitization, the work will be briefly described.

Light from a quartz mercury-arc lamp was passed into a reaction tube contained in a cylindrical, electrically heated jacket whose end was closed by a quartz lens. The jacket was ordinarily kept at 45° as in the experiments of Cario and Franck; the reaction was, however, also found to go at room temperature. The quartz reaction tube, 2 cm. in diameter and 16 cm. long, had a clear quartz window fused to one end and a small quartz tube joined to the other. A sealing-wax joint on the latter connected the reaction tube with a narrow, glass U-tube immersed to a constant depth in liquid air. The total volume of the gas space was about 160 cc. A side bulb containing mercury which could be drawn out permitted the gas to to be expanded and recompressed facilitating the removal of condensible material by the liquid air. Through stopcocks the gas space could be placed in communication with either a mercury diffusion pump or a chamber containing a suitable gas reaction mixture. The hydrogen and oxygen used were produced by the electrolysis of a barium hydroxide solution. A strong blast of air on the mercury lamp was held fairly constant with the aid of a Pitot tube.

The experiments were made in the following manner: after pumping out the reaction tube system, a definite gas mixture was admitted and the pressure read. With a definite current passing through the lamp, the reaction tube was illuminated for a period (usually 10 minutes) timed with a stop watch. After the removal of any condensible material, the pressure was again read. This process was usually repeated a number of times.

The results of a typical series of experiments, numbered in the order made, are shown in the figure; the total pressure is plotted against the total

time of illumination. One or two cc. of fresh mercury were placed in the reaction tube at the beginning of the series and remained through Experiment IV. In all experiments except parts of IV, the blast of air was maintained on the lamp. Experiment I was a blank made with pure hydrogen. In II, a mixture of 46 mol % hydrogen and 54 mol % oxygen was used. Experiment III was a blank made using pure oxygen. IV



differed from II only in having no air blast in the first 30 and last 20 minutes. V differed from II only in having no mercury present in the reaction tube. The rate of decrease of pressure in experiments like V seemed to depend greatly on the care used to remove all traces of mercury from the reaction tube; in some trials almost no decrease was observed.

In the above experiments a considerable rate of pressure decrease was obtained only when oxygen, hydrogen, and mercury were all present in the reaction tube and an air-blast was maintained on the lamp. In the absence of any one of the substances, or of a radiation capable of exciting gaseous mercury atoms (as when the air-blast was interrupted), the rate was comparatively small or zero. It seems clear, therefore, that excited mercury atoms can bring about the combination of hydrogen and oxygen when the latter is present in the elementary state. These results are in agreement with those of Cario and Franck on metal oxides.

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[1] G. Cario and J. Franck, *Zt. Physik*, **11**, 161, 1922.